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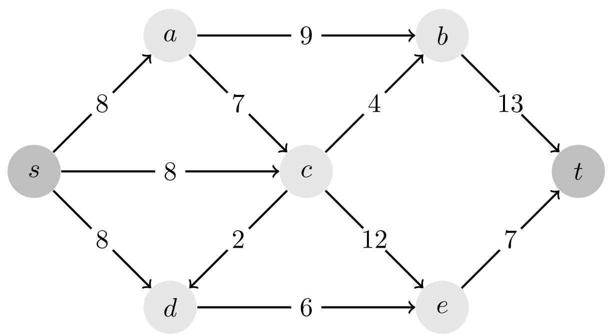
Algorithms and Probability **Session Sheet S11** **FS 2026**

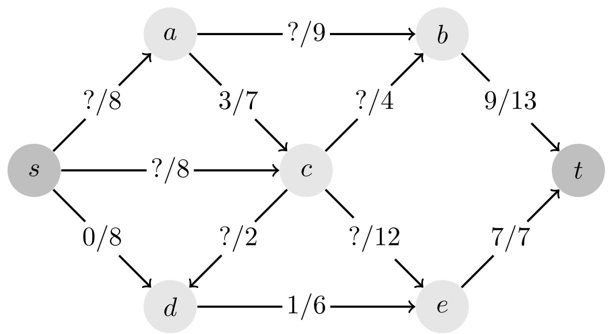
For the exercise sessions on 07 May 2026.

Exercise S11.1 – *Flows*

Consider the following network N. The number at each edge represents its capacity.



(a) Fill in the missing values so that they form a feasible (not necessarily maximum) flow:



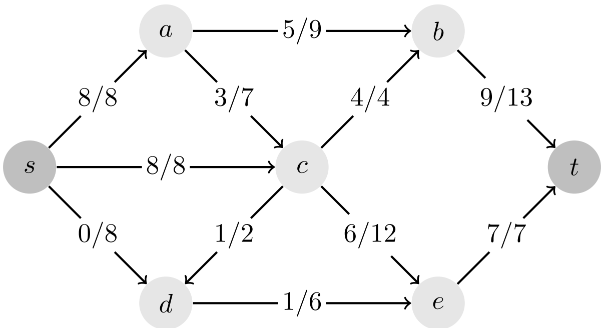
(b) Construct the residual network ("Restnetzwerk").

(c) Find an augmenting s-t-path and augment the flow along this path. If necessary, repeat this step until you have found a maximum flow.

(d) Prove that your flow is a maximum flow by finding a minimum cut in N.

Solution S11.1 – *Flows*

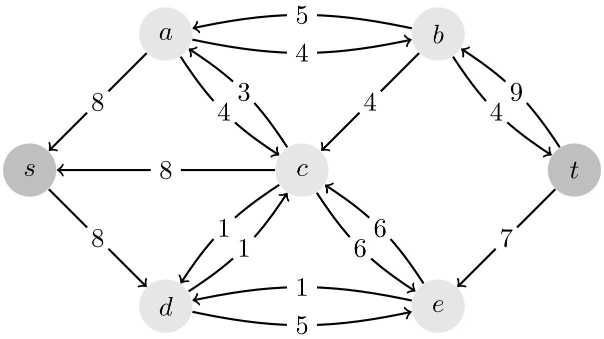
(a) The correct flow looks as follows:



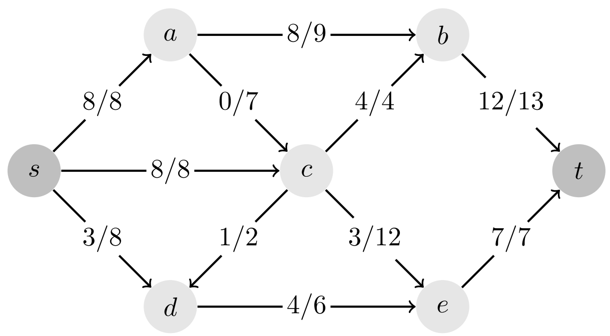
There are different ways to compute the remaining values. We give one possible line of arguments:

- Due to flow conservation in d, the edge (c, d) has flow 1.
- Due to flow conservation in e, the edge (c, e) has flow 6.
- Because the net outflow of s equals the net inflow of t, the edges (s, a), (s, c), and (s, d) together need a flow of $7 + 9 = 16$. Since the flow on (s, d) is zero, (s, a) and (s, c) together have a flow of 16. Due to their capacity, this can only be realized by putting value 8 on both edges.
- Due to flow conservation in a, the edge (a, b) has flow 5.
- Due to flow conservation in b, the edge (c, b) has flow 4.

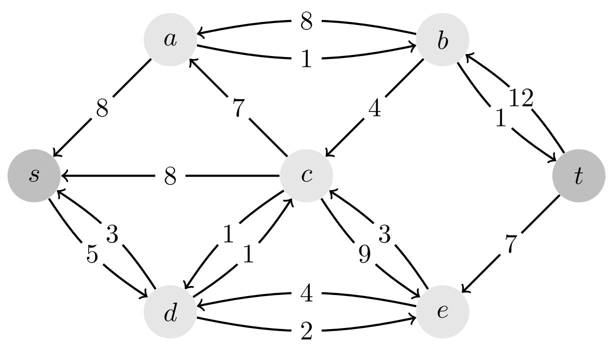
(b)



(c) A possible augmenting path is (s, d, e, c, a, b, t). Along this path, we can increase the flow by 3. This is because the residual capacity of (c, a) is 3 (in other words, we can decrease the flow along the edge (a, c) by at most 3). After augmenting, we obtain the following flow:



The value of the new flow equals 19 (which we can determine by computing the net outflow of s or the net inflow of t). Its residual network looks as follows and does not contain an s-t-path.



(Remark: we could have chosen the path (s, d, c, a, b, t) instead. In this case, we would need to find another augmenting path afterwards.)

(d) To find a minimum cut, we can consider all vertices reachable from s in the residual network. This gives us the cut $S = \{s, c, d, e\}$ and $T = \{a, b, t\}$, which is an s-t-cut (S, T) with capacity $c(s, a) + c(c, b) + c(e, t) = 8 + 4 + 7 = 19$. Because $\text{val}(\tilde{f}) = 19 = \text{cap}(S, T)$, Theorem 3.9 (Maxflow-Mincut Theorem) implies that both our flow and our cut are optimal.